

Recent developments in string theory : a brief review for particle physicists

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Abstract : At the present time, string theory (and its generalizations) remain relatively abstruse subjects to the particle phenomenologist and experimentalist. Yet, striking developments of the last two years offer hope that a fundamental non-perturbative formulation of this theory will be found, and that this formulation will permit us to make contact with supersymmetric standard-model physics. This article is based on a talk which attempted to convey the essence of these recent developments in string theory, in a non-technical manner, to an audience of particle theorists, phenomenologists and experimentalists.

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1. Introduction : historical perspective

The theory of relativistic quantized strings became a compelling candidate for the unification of elementary interactions, after a number of remarkable properties were uncovered. This process took over a decade, at the end of which (around 1983) one could make the following list of properties¹ :

- At low energies, string theory is described by a supersymmetric quantum field theory containing gravitons, gauge particles and fermions in a unified framework.
- At high energies, it differs from ordinary field theory, and is ultraviolet finite *despite* the presence of gravitons.
- There is no coupling constant. Instead, there is a massless scalar field, the “dilaton”, whose (perturbatively undetermined) VEV acts as the gauge and gravitational coupling.

¹For all references to these results of the pre-duality era, see Ref. [1]

- Supersymmetry seems to be *required* for consistency.
- The total number of spacetime dimensions is 10, although as in any theory containing gravity, each of these dimensions could be compact or noncompact. A *choice of vacuum configuration* is required for a physical interpretation as spacetime, and this could have any number $D \leq 10$ of noncompact spacetime dimensions.

Remarkably, this list of properties failed to cause large-scale enthusiasm in the theoretical particle physics community. But unprecedented excitement arose in 1984 when one more impressive and rather unexpected property was added to this list. The superstrings with $N = 1$ supersymmetry in 10 dimensions, which are the ones possessing Yang-Mills gauge fields, are chiral and hence potentially anomalous at the quantum level. It was shown that they are in fact free of gauge and gravitational anomalies for just two choices of the gauge group: $SO(32)$ and $E_8 \times E_8$.

This sparked off a wave of interest in string theory, leading to the discovery of a whole new set of properties :

- The superstring with $E_8 \times E_8$ gauge symmetry, the “heterotic string”, admits consistent vacuum solutions of the form of a compact 6-manifold, times flat 4D Minkowski spacetime. A natural class of such models give $N = 1$ supersymmetry in 4D. This is only a quarter as much SUSY as $N = 1$ in 10D, as the 6-manifold background—a “Calabi-Yau” space—breaks 3/4 of the original supersymmetries.
- This class of models easily admits chiral fermion representations in 4d. A simple pattern of symmetry-breaking is

$$E_8 \times E_8 \rightarrow E_8 \times E_6 \quad (1)$$

with fermions in the 27 of E_6 . Alternatively E_6 could be replaced by $SO(10)$ with fermions in the 16.

The net number of chiral fermion generations is related to the topology of the Calabi-Yau 6-manifold for example,

$$n_{27} - n_{\bar{27}} = \frac{1}{9} |\chi| \quad (2)$$

where χ is the Euler characteristic of the manifold.

- In this scenario, one entire E_8 is naturally unbroken, but communicates only gravitationally with the standard gauge sector. Thus it could plausibly be interpreted as a “hidden sector” consisting of dark matter.

The above picture is grossly oversimplified in many ways. Compactifications far more general than the type described above are equally possible, but require more

complicated techniques for their analysis. More important, the above results did not provide much of a clue about what should be done to give the 4-dimensional theory a truly realistic spectrum of particles : this would require supersymmetry to be broken, and both masses and mass splittings to be generated. The dilaton would need to be stabilized by the generation of a potential.

In addition, there was one problematic prediction in the heterotic string. This can be shown in various ways, though we will use a simple, classical scaling argument due to Witten [2]. The Planck scale in a D -dimensional theory of gravity is defined (upto constants of order 1) by writing the action as

$$S_D[g] = (M_D^{pl})^{D-2} \int d^D x \sqrt{g} R \quad (3)$$

where the power of M_D^{pl} makes the action dimensionless, given that the action is built from dimensionless fields, 2 derivatives and D integrations. In 10-dimensional string theory the low-energy effective action is also commonly written

$$S_{10}[g] = (M_s)^8 \int d^{10} x e^{-2\phi} \sqrt{g} R \quad (4)$$

where M_s is by definition the string scale, and ϕ is the dilaton, related to the string coupling by $g_s = e^\phi$. From this it is evident that the 10d Planck mass in string theory satisfies $(M_{10}^{pl})^8 = (g_s)^{-2} (M_s)^8$.

Now if we dimensionally reduce the above actions on a 6-dimensional manifold of volume V (assumed to be isotropic), the volume factors out and the effective 4d action can be written :

$$S_4[g] = V (M_{10}^{pl})^8 \int d^4 x \sqrt{g} R = (M_4^{(pl)})^2 \int d^4 x \sqrt{g} R \quad (5)$$

From the above equations, the 4-dimensional Planck mass is related to the string parameters M_s , g_s , V by

$$M_4^{(pl)} \sim \sqrt{V} \cdot (M_s)^4 \quad (6)$$

A similar calculation can now be carried out for the gauge interactions arising from string theory. Here, we require that the gauge fields have canonical dimension 1 in 10 dimensions, so that upon dimensional reduction to 4 dimensions they will automatically have the canonical dimensions appropriate to 4d gauge theory. Thus the string low-energy action in 10d has a term²

$$S_{10}[A] = (M_s)^6 \int d^{10} x e^{-2\phi} \sqrt{g} \text{tr} F_{\mu\nu} F^{\mu\nu} \quad (7)$$

²The dilaton-dependence in this term is specific to the heterotic string.

After compactification, the 4d action will now be interpreted as the GUT action for the gauge theory, so it is written

$$\begin{aligned} S_4[A] &= V \frac{(M_s)^6}{(g_s)^2} \int d^4 x \sqrt{g} \operatorname{tr} F_{\mu\nu} F^{\mu\nu} \\ &= \frac{1}{(\alpha_{\text{GUT}})^2} \int d^4 x \sqrt{g} \operatorname{tr} F_{\mu\nu} F^{\mu\nu} \end{aligned} \quad (8)$$

from which we get our second equation, relating the 4d GUT coupling constant to the string parameters :

$$\frac{1}{\sqrt{\alpha_{\text{GUT}}}} \sim \sqrt{V} \frac{(M_s)^3}{g_s} \quad (9)$$

To get a prediction about the real world, we must eliminate the string scale M_s between Eqs. (6) and (9), and set the volume V of the internal space to be given by the GUT mass scale : $V \sim (M_{\text{GUT}})^{-6}$. Then we find :

$$M_4^{(pl)} \sim (g_s)^{1/3} \frac{M_{\text{GUT}}}{\alpha_{\text{GUT}}^{2/3}} \quad (10)$$

For weakly couple string theory, the above relation implies

$$M_4^{(pl)} \ll \frac{M_{\text{GUT}}}{\alpha_{\text{GUT}}^{2/3}} \quad (11)$$

which is unacceptably small. This is the gauge coupling unification problem in string theory.

While many ways have been proposed to overcome this problem (see for example Ref. [3] and references therein), it is clear that the problem goes away if we are not committed to weakly coupled string theory. In other words, if we know something about the strongly coupled, nonperturbative regime in string theory, then eq. (10) above can simply be used as a phenomenological input to determine what is the (large) value of the string coupling relevant to the real world.

Recent developments in string theory have made this seem less of an impossibility, and we will return to this point.

2. Dualities in string theory

Two kinds of duality have played a crucial role in enlarging our understanding of string theory. I will briefly review how they operate.

The first duality that we encounter is called “target-space duality” or “T-duality”. This is an essentially stringy phenomenon, but can be understood in a non-technical way from a simple picture.

Consider a spacetime containing one direction compactified on a circle of radius R . A point particle propagating on the circle has quantized momenta $p \sim \frac{n}{R}$, as we know from elementary quantum mechanics. A string propagating on the same circle can behave like a point particle located at its centre of mass (just by becoming a very small string), so string theory too has quantized momentum modes on a circle, with energies $E \sim \frac{n}{R}$.

But the string can do something new : it can “wrap” itself on the circle an integer number of times, creating a “winding mode”. These modes have an energy proportional to the radius, simply because the energy in an extended string is equal to its tension (a constant of string theory) times its length. Thus for such modes $E \sim m(M_s)^2 R$ where m is some integer, and the string scale has been inserted for dimensional reasons. The two kinds of modes are depicted in this figure :

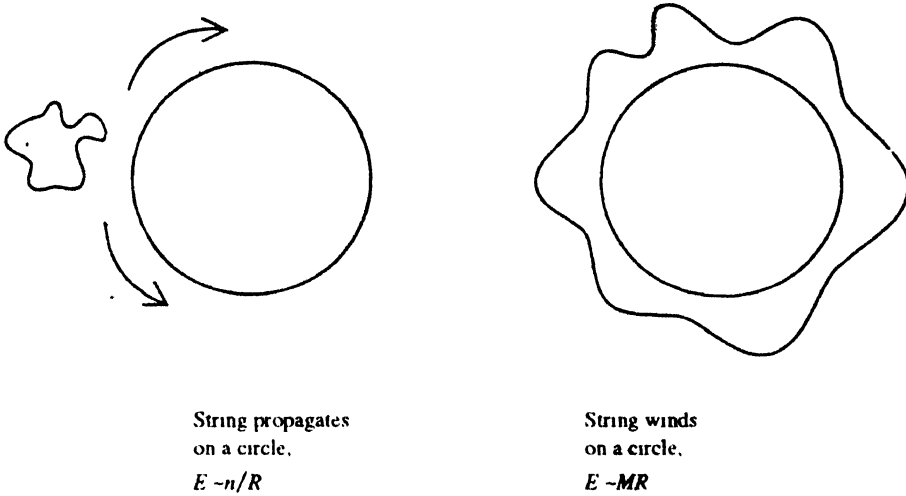


Figure 1. Duality between momentum and winding modes.

Now, the replacement

$$M_s R \rightarrow \frac{1}{M_s R} \quad (12)$$

interchanges the spectra of the string momentum and winding modes. Since this operation can be carried out at weak coupling, one can ask if it is a symmetry of perturbative string theory. Indeed, it can be shown to be so (we will not do it here).

Thus, in string theory, short (sub-Planckian) distances are equivalent to long distances. As one consequence, a string theory compactified on a very small circle will be equivalent to one on a very large circle, hence the number of (approximately) noncompact spacetime dimensions in a given situation is not well-defined. The winding modes on a

small circle behave like momentum modes on a large one. Our physical interpretation, based on particles, would probably be in terms of the larger number of dimensions.

Another duality, for which the arguments are by no means so rigorous, is “strong-weak duality” or “S-duality”. This is the statement that, in some cases, weakly coupled string theory with string coupling g_s is equivalent to strongly coupled string theory with coupling $\frac{1}{g_s}$. As with T-duality, different types of modes get interchanged by the S-duality operation. In some situations (typically in compactifications to four spacetime dimensions) it exchanges electrically charged with magnetically charged excitations of the theory.

More generally, strong-weak duality exchanges “fundamental” with “solitonic” modes in string theory. The former are perturbative while the latter are visible in the spectrum as non-trivial solutions of the classical equations of motion. (More details about this were given in the lecture of A. Dabholkar at this Symposium.)

Let us investigate S-duality a little more explicitly, in the most basic situation where it appears. This is the so-called type IIB superstring in 10 dimensions, whose low-energy Lagrangian is type IIB (*i.e.* chiral, $N = 2$) supergravity. The spectrum of massless fields in this supergravity is dictated purely by classical considerations. The only irreducible supermultiplet is that of supergravity, with bosonic fields as follows :

$$\begin{array}{ll}
 g_{\mu\nu} & : \text{metric} \\
 B_{\mu\nu} & \text{2-form field} \\
 B_{\mu\nu} & \text{another 2-form field} \\
 \phi & \text{scalar (dilaton)} \\
 \tilde{\phi} & \text{another scalar (axion)} \\
 \tilde{D}_{\mu\nu\lambda\rho} & \text{4-form field}
 \end{array} \tag{13}$$

Here by “ p -form field” we mean a bosonic field transforming as a totally antisymmetric p -th rank tensor. This multiplet contains fermions too, but we will not need to list them.

It is convenient to define the complex scalar field $\tau \equiv \bar{\phi} + ie^{-\phi}$. The low-energy Lagrangian, in the approximation of slowly-varying fields (no more than two derivatives), is dictated by supersymmetry and turns out to be invariant under

$$\frac{a\tau + b}{c\tau + d}, \quad \begin{array}{cc} a & c \\ b & d \end{array} \tag{14}$$

where a, b, c, d are real numbers satisfying $ad - bc = 1$. These transformations form the group $SL(2, R)$.

Although this is a symmetry of the low-energy effective Lagrangian, it certainly cannot be an exact symmetry of string theory. The reason is that string theory has states carrying *charge* under the 2-form fields B, \tilde{B} . Indeed, such states are strings : in general, an

object extended in $p-1$ spatial directions carries charge under p -forms. The familiar case is $p = 1$ for which we get particles charged under gauge fields. The type IIB string itself carries unit charge under $B_{\mu\nu}$, while there is a certain solitonic string carrying unit charge under $\tilde{B}_{\mu\nu}$. $SL(2, R)$ would not only mix up these charges but in general make them fractional (even irrational) which is forbidden by charge quantization.

The largest subgroup of $SL(2, R)$ which can conceivably be an exact symmetry of the IIB string is $SL(2, Z)$, consisting of matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ with a, b, c, d integral and $ad-bc = 1$. This preserves integrality of charges, but, from eq. (14), rotates the B and \tilde{B} fields into each other, hence also the fundamental into the solitonic string — as promised.

A large and very tightly constrained set of results leads us to believe that $SL(2, Z)$ is indeed an exact symmetry of the type IIB string. In particular, the choice of transformation

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \text{ corresponds to the transformation} \quad \tau \rightarrow -\frac{1}{\tau} \quad (15)$$

which, if $\tilde{\phi} = 0$, corresponds to

$$e^{\phi} \rightarrow e^{-\phi} \quad (16)$$

or in other words, $g_s \rightarrow g_s^{-1}$. Thus the S-duality group $SL(2, Z)$ contains strong-weak duality as a Z_2 subgroup. The non-Abelian nature of the full group, however, makes it still more interesting.

Although not strictly related to the theme of this talk, this stringy duality has some amazing implications for ordinary (supersymmetric) quantum field theory without gravity. In recent years, it has been argued with considerable difficulty and ingenuity [4,5] that $N = 4$ supersymmetric Yang-Mills (SYM) theory in $3 + 1$ dimensions has exact electric-magnetic $SL(2, Z)$ duality symmetry. It has also been argued [6] that $N = 2$ SYM field theory can be solved nonperturbatively, and that this solution makes use of $SL(2, Z)$ duality transformations in an essential way.

Both these facts, superficially unrelated to each other and to 10-dimensional string theory, are actually consequences of the $SL(2, Z)$ duality of the type IIB string that we have discussed above. Type IIB string theory contains various extended objects including “3-branes” and “7-branes” (similar to the membranes or domain walls studied in cosmological theories, but one and five dimensions higher respectively). It turns out that the dynamics of an isolated 3-brane is governed by an $N = 4$ SYM field theory, which inherits $SL(2, Z)$ duality from the string theory in which it is embedded. Similarly, the dynamics of a 3-brane parallel to, and near, a 7-brane, is governed by an $N = 2$ SYM field theory, whose nonperturbative solution, including the duality transformations, is inherited from the type IIB string [7,8].

One lesson from this is that nontrivial properties of flat-spacetime field theories become more transparent when these theories are viewed as sectors of string theory. Even if string theory is not the right unified theory of nature, it could well be an essential new way to understand quantum field theories!

3. Beyond string theory : “M-theory”

We have seen that the type IIB string has a strong-weak-coupling S-duality. This means that its dynamics at strong coupling is understood, and can be extracted by mapping onto a weakly coupled dual theory. However, an analogous result cannot hold for the type IIA string, whose low-energy limit is non-chiral $N = 2$ supergravity. The low energy effective action in 10 spacetime dimensions does not admit any symmetry which includes interchange of strong and weak coupling. So a new insight is needed.

In the old days, extended supergravity theories were often constructed by compactifying higher dimensional theories on a circle (the “Kaluza-Klein” procedure). For certain values of D , spinors of the Lorentz algebra split into two when we go down from D to $D-1$ dimensions. In these cases, compactification of a supersymmetric theory doubles the number of spinor supercharges. Moreover, if D is odd, then the parent theory is obviously non-chiral, and reduction leaves us with a non-chiral theory in even dimensions.

In 11 spacetime dimensions, there is a unique classical supergravity theory, which has $N = 1$ supersymmetry. Upon Kaluza-Klein reduction on a circle, it gives precisely the type IIA supergravity in $D = 10$. The 11-dimensional theory possesses no scalar particle analogous to the dilaton of string theory, hence it has no dimensionless coupling constant. But after circle compactification, one component of the metric becomes such a scalar. By comparing classical actions in 10D and 11D, it is easy to obtain the following relations between 11-dimensional quantities (the Planck mass $M_{11}^{(Pl)}$ and the compactification radius R_{11}) and 10-dimensional quantities (the string scale and the string coupling) :

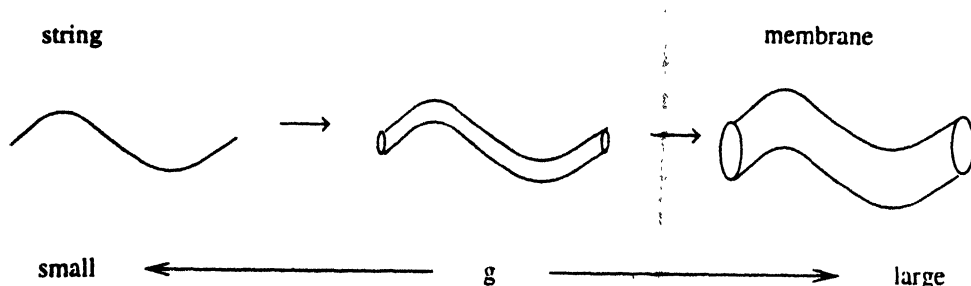
$$\begin{aligned} M_{11}^{(Pl)} &= \frac{M_s}{(g_s)^{\frac{1}{3}}} \\ R_{11} &= \frac{g_s}{M_s} \end{aligned} \tag{17}$$

Now if we choose units for which $M_{11}^{(Pl)} = 1$ then we find

$$R_{11} = (g_s)^{\frac{2}{3}} = e^{\frac{2}{3}\phi} \tag{18}$$

Classically, this seems to say that type IIA supergravity in $D = 10$ becomes 11-dimensional at strong coupling. But there is considerable evidence for a much stronger conjecture : that the strong-coupling limit of *quantized* type IIA *string* theory, is some “new” 11-dimensional quantum theory, having 11-dimensional supergravity as its low-energy limit. This new theory is called “M-theory” [9,10].

Thus, M-theory is more than just supergravity in $D = 11$. For example, it contains membranes in its spectrum. Their presence explains the relation to string theory : the type IIA string is believed to be just the membrane of M-theory, wrapped round the circle of radius R_{11} . This looks like a string for small values of R_{11} , which by eq. (18) above, is just the limit of weakly coupled string theory where we indeed expect to see the fundamental string. The situation looks as follows :



Some people believe that M-theory is a theory of fundamental membranes, rather than strings, but this has yet to be convincingly demonstrated. It has, however, very convincingly been argued that M-theory is a sensible quantum theory containing various extended objects. The very existence of this theory implies many important properties of string theory. For example, the type IIA string is known to contain others extended objects besides strings, the “ p -branes”, and these can be argued to arise from M-theory [11].

Even more impressive, one can relate M-theory to the type IIB string, though the latter is a chiral theory. The relation is that M-theory compactified to 9 dimensions on a 2-torus, in the limit that the torus shrinks, is the 10-dimensional type IIB string! To argue this equivalence [11,12], we make a T-duality on the way, which interchanges the type IIA and type IIB strings. The apparent paradox inherent in equating a 9-dimensional with a 10-dimensional theory is resolved by the observation, made earlier, that T-duality in string theory permits precisely this.

As a consequence, we can *predict* the nonperturbative $SL(2, Z)$ S-duality of the type IIB string. It is just the geometrical $SL(2, Z)$ symmetry group of the 2-torus on which M-theory is compactified! Strong-weak coupling interchange is just exchange of the two cycles of the torus. Thus M-theory plays a powerful role in geometrizing the dualities of string theory.

4. M-theory and the heterotic string

So far we have spoken of M-theory as an 11-dimensional theory underlying the type IIA and IIB strings. But these are not the string theories having the most promising connection with particle physics. As we discussed in the beginning, it is the $E_8 \times E_8$ heterotic string which has the most plausible relationship to grand unification. I will now argue that the $E_8 \times E_8$ heterotic string can also be derived from M-theory [13].

Although M-theory is less well-understood than string theory, it has some “stringy” properties. Among these is the fact that it can be compactified not only on smooth manifolds (which elementary particle theories require) but also on singular ones called orbifolds. An orbifold is an almost-everywhere smooth manifold with some specific types of singularities. It can be described as the quotient M/Γ where M is a smooth manifold (typically a torus) and Γ is a discrete group of symmetries of M . The quotient space has singularities wherever the quotienting group has fixed points on the manifold.

The spectrum of strings on an orbifold M/Γ is related to that on M . First of all, clearly we must keep string configurations on M which are invariant under Γ . But additionally, we should include string configurations which would not have been well-defined on M , but are well-defined after quotienting with Γ . The latter are called “twisted sector states”.

The simplest orbifold possible comes from the choice $M = S^1$, a circle, and $\Gamma = \mathbb{Z}_2$. Let the circle coordinate x take the range $0 \leq x \leq 2\pi R$, and let the \mathbb{Z}_2 action send x to $-x$. The quotient space is the interval $0 \leq x \leq \pi R$ with two end-points. Compactifying M-theory to 10 dimensions on this simple orbifold leads to a remarkable surprise.

It can be shown that the \mathbb{Z}_2 action projects out half the gravitinos that M-theory would have had if compactified just on a circle to 10D. Thus it has only $N = 1$ supersymmetry rather than $N = 2$. Indeed, the \mathbb{Z}_2 projection leaves us with the $N = 1$ supergravity multiplet as the “untwisted” sector. If there is an analogue of a twisted sector, then this is restricted by supersymmetry, which admits only a super-Yang-Mills (SYM) multiplet besides the supergravity multiplet. So without any computation, we conclude that the twisted sector, if it gives anything, contributes Yang-Mills fields and hence gauge symmetry to the orbifold theory.

With our limited understanding of M-theory it would be hard to predict the actual gauge group, but for one happy circumstance. The $N = 1$ supergravity multiplet in 10D is chiral and has a gravitational anomaly. Therefore, to get a consistent theory we must find a way of cancelling the anomaly using the twisted sector states.

Now, it has long been known that gravitational, gauge and mixed anomalies can be cancelled in $N = 1$ supergravity in 10D only if the gauge group is one of $SO(32)$ or $E_8 \times E_8$. But in our case the anomalies are located at the two ends of the interval $0 \leq x \leq \pi R$ (in between, the theory is effectively 11-dimensional and cannot have gravitational anomalies). So local cancellation of anomalies requires that the gauge group appear in two factors, one associated to each end of the interval. That uniquely fixes the gauge group, from among the two choices above, to be $E_8 \times E_8$.

This leads to the conjecture that M-theory compactified on S^1/\mathbb{Z}_2 is actually the heterotic string with gauge group $E_8 \times E_8$. This immediately brings M-theory into prominence in attempts to extract particle physics as a low-energy limit of string theory. When the string is strongly coupled, an 11th dimension, in the form of an interval with two end-points, opens up.

More general orbifolds of M-theory also exist [14–18], and should play an important role in providing more general classes of M-theory compactification to 4 dimensions.

5. An M-theory application to unification physics

With this result, the strong coupling behaviour of the heterotic string is no longer a mystery. At strong coupling, we replace the perturbative string description (which is no longer appropriate) by the description as M-theory on 10D spacetime times an interval S^1/Z_2 . The length of the interval is proportional to a power of the string coupling, exactly as in eq. (18).

The coupling unification problem that was described in Section 1 now admits an obvious solution. We are no longer committed to the weakly coupled heterotic string, so the undesirable bound Eq. (11) need not hold. Instead, one can use eqs. (9), (10) and (17) to calculate the radius of the 11th dimension. One finds [2] :

$$M_4^{(pl)} \sim \frac{R_{11}^{\frac{1}{2}} M_{\text{GUT}}^{\frac{3}{2}}}{\alpha_{\text{GUT}}^{\frac{3}{4}}} \quad (19)$$

from which the radius of the eleventh dimension is

$$R_{11} \sim \frac{\left(M_4^{(pl)}\right)^2 \alpha_{\text{GUT}}^{\frac{3}{2}}}{M_{\text{GUT}}^3} \quad (20)$$

A more careful calculation leads to an estimate of around 80 for the numerical coefficient on the RHS of eq. (19), hence the RHS of eq. (20) actually has a prefactor as small as 1.5×10^{-4} .

A detailed analysis of this and other fascinating results in the literature is beyond the scope of the present article. It should be noted that the present scenario is far from complete or convincing, for a variety of reasons (see, for instance, the very detailed discussion in ref. [19]). The purpose of exhibiting these simple calculations here is to illustrate how problems that were virtually impossible to address in perturbative string theory, have been opened up for discussion with the advent of string dualities and M-theory.

6. Conclusions

String theory remains the prime candidate for a unified theory of all fundamental interactions. Recent developments have given us unexpected control over some nonperturbative and strong-coupling effects in string theory. Moreover, it appears that whatever we formerly knew about string theory, plus much more, can be subsumed into a more general framework called “M-theory”.

A modest literature has grown up over the proposal that 4-dimensional unification physics can be obtained from compactifications of M-theory, either of the type $S^1/Z_2 \times M_6$ discussed above, or directly via more general orbifolds [17,18]. The interested reader may

consult the review by Dienes [20] and the papers in Ref. [21] (this is only a partial list of recent papers).

Due to a shortage of spacetime, an important and complementary development called “F-theory” [22] has been left out of the present discussion. F-theory offers another approach to study nonperturbative string physics after compactification to 4 dimensions. After years of having no control over strong-coupling behaviour in compactified string theories, we now have at least two different approaches! It remains to be seen how long it takes to assemble all this into genuine progress towards the goal of unification.

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